



O-notation +
master theorem



O-notation

$$f(n) = O(g(n)) \Leftrightarrow$$

$$\Leftrightarrow \exists c \in R, \exists n_0 \in N : f(n) \leq cg(n), \forall n \geq n_0$$

$$f(n) = \Omega(g(n)) \Leftrightarrow$$

$$\Leftrightarrow \exists c \in R, \exists n_0 \in N : f(n) \geq cg(n), \forall n \geq n_0$$

$$f(n) = \Theta(g(n)) \Leftrightarrow$$

$$\Leftrightarrow \exists c_1, c_2 \in R, \exists n_0 \in N : c_1 g(n) \leq f(n) \leq c_2 g(n), \forall n \geq n_0$$



O-notation (2)

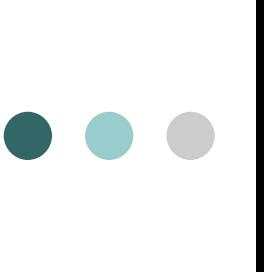
Παρατηρήσεις: $f(n)=O(g(n)) \Leftrightarrow g(n)=\Omega(f(n))$

$f(n)=\Theta(g(n)) \Leftrightarrow f(n)=O(g(n)) \wedge f(n)=\Omega(g(n))$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f(n)=O(g(n))$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty \Rightarrow f(n)=\Omega(g(n))$$

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = c \in R^+ \Rightarrow f(n)=\Theta(g(n))$$

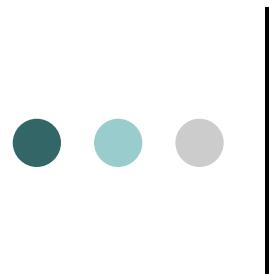


Ασυμπτωτικός τύπος Stirling

$$n! \approx e^{-n} n^{n+0.5} \sqrt{2\pi}$$

$$\lim_{n \rightarrow \infty} \frac{n!}{e^{-n} n^{n+0.5}} = \sqrt{2\pi}$$

$$n! = \sqrt{2\pi n} \left(\frac{n}{e} \right)^n \left(1 + \Theta(1/n) \right)$$



Master theorem

Αν $a \geq 1, b > 1$ και $T(n) = aT(n/b) + f(n)$
έχουμε:

- 1) $\exists \varepsilon > 0 : f(n) = O(n^{\log_b a - \varepsilon}) \Rightarrow T(n) = \Theta(n^{\log_b a})$
- 2) $f(n) = \Theta(n^{\log_b a}) \Rightarrow T(n) = \Theta(n^{\log_b a} \log n)$
- 3) $\exists \varepsilon > 0 : f(n) = \Omega(n^{\log_b a + \varepsilon})$ και $af(n/b) \leq cf(n)$
για $c < 1$ και μεγάλο $n \Rightarrow T(n) = \Theta(f(n))$