O-notation + master theorem

O-notation

$$f(n) = O\left(g(n)\right) \Leftrightarrow$$
$$\Leftrightarrow \exists c \in R, \exists n_0 \in N : f(n) \le cg(n), \forall n \ge n_0$$

$$f(n) = \Omega\left(g(n)\right) \Leftrightarrow$$
$$\Leftrightarrow \exists c \in R, \exists n_0 \in N : f(n) \ge cg(n), \forall n \ge n_0$$

$$f(n) = \Theta\left(g(n)\right) \Leftrightarrow$$
$$\Leftrightarrow \exists c_1, c_2 \in R, \exists n_0 \in N : c_1 g(n) \le f(n) \le c_2 g(n), \forall n \ge n_0$$

O-notation (2)

Παρατηρήσεις: $f(n)=O(g(n)) \Leftrightarrow g(n)=\Omega(f(n))$

$$f(n)=\Theta(g(n)) \Leftrightarrow f(n)=O(g(n)) \wedge f(n)=\Omega(g(n))$$

$$\lim_{n\to\infty}\frac{f(n)}{g(n)}=0 \Rightarrow f(n)=O(g(n))$$

$$\lim_{n\to\infty}\frac{f(n)}{g(n)}=\infty \Rightarrow f(n)=\Omega(g(n))$$

$$\lim_{n\to\infty}\frac{f(n)}{g(n)}=c\in R^{+} \Rightarrow f(n)=\Theta(g(n))$$

Ασυμπτωτικός τύπος Stirling

$$n! \approx e^{-n} n^{n+0.5} \sqrt{2\pi}$$

$$\lim_{n \to \infty} \frac{n!}{e^{-n} n^{n+0.5}} = \sqrt{2\pi}$$

$$n! = \sqrt{2\pi n} \left(\frac{n}{e} \right)^{n} \left(1 + \Theta(1/n) \right)$$

Master theorem

Αν $a \geq 1, b > 1$ και $T(n) = aT(n/b) + f(n)$ έχουμε:

1) $\exists \varepsilon > 0 : f(n) = O\left(n^{\log_b a - \varepsilon}\right) \Rightarrow T(n) = \Theta\left(n^{\log_b a}\right)$

2) $f(n) = \Theta\left(n^{\log_b a}\right) \Rightarrow T(n) = \Theta\left(n^{\log_b a} \log n\right)$

3) $\exists \varepsilon > 0 : f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$ και $af(n/b) \leq cf(n)$

για $c < 1$ και μεγάλο n $\Rightarrow T(n) = \Theta\left(f(n)\right)$