Mechanism Design without Money

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Viewpoint shaped through joint work with **Christos Tzamos**

Social Choice

Setting

- Set A of possible **alternatives** (candidates) .
- Set $N = \{1, \ldots, n\}$ of **agents** (voters).
- $\forall$ agent i has a (private) **linear order** $\succ_i \in L$ over alternatives A.

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Desirable Properties of Social Choice Functions

- Onto : Range is A.
- Unanimous : If a is the top alternative in all $\succ_1, \ldots, \succ_n$, then
$$F(\succ_1, \ldots, \succ_n) = a$$
- Not dictatorial : For each agent i, $\exists \succ_1, \ldots, \succ_n$:
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- **Strategyproof** or **truthful** : $\forall \succ_1, \ldots, \succ_n, \forall$ agent $i, \forall \succ'_i$,
$$F(\succ_1, \ldots, \succ_i, \ldots, \succ_n) \succ_i F(\succ_1, \ldots, \succ'_i, \ldots, \succ_n)$$

Impossibility Result

Gibbard-Satterthwaite Theorem (mid 70's)

Any **strategyproof** and **onto** social choice function on **more than 2** alternatives is **dictatorial** .

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- Randomization
- Monetary payments
- Voting systems **computationally hard** to manipulate.

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- Randomization
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- Voting systems **computationally hard** to manipulate.
- **Restricted domain** of preferences – **Approximation**

Single Peaked Preferences and Medians

Single Peaked Preferences

- One dimensional ordering of alternatives, e.g. $A = [0, 1]$
- Each agent i has a **single peak** $x_i^* \in A$ such that for all $a, b \in A$:

$$b < a \leq x_i^* \quad \Rightarrow \quad a \succ_i b$$
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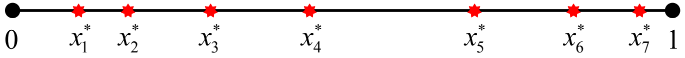

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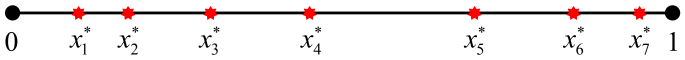
Median Voter Scheme [Moulin 80], [Sprum 91], [Barb Jackson 94]

A social choice function F on a single peaked preference domain is **strategyproof**, **onto**, and **anonymous** iff there exist $y_1, \ldots, y_{n-1} \in A$ such that for all $(x_1^*, \ldots, x_n^*)$,

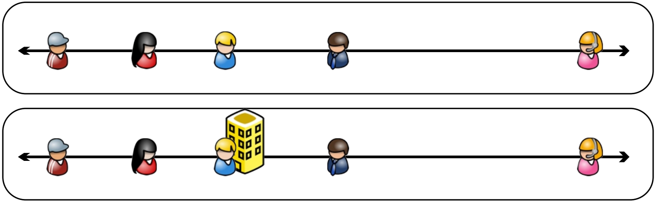
$$F(x_1^*, \ldots, x_n^*) = \text{median}(x_1^*, \ldots, x_n^*, y_1, \ldots, y_{n-1})$$



Single Peaked Preferences and Medians

Select a Single Location on the Line

The **median** of $(x_1, \ldots, x_n)$ is **strategyproof** (and Condorcet winner) .



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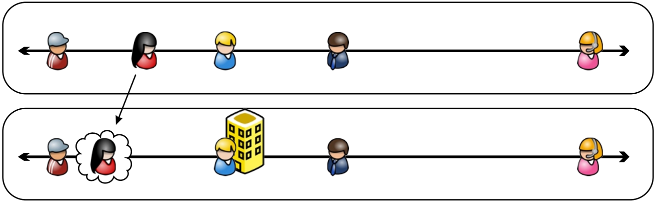

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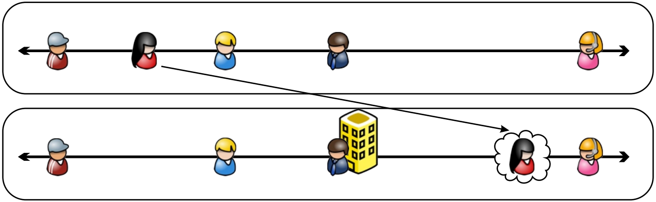
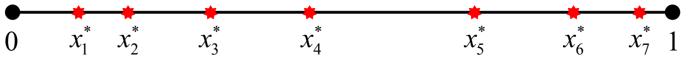


Single Peaked Preferences and Generalized Medians

Generalized Median Voter Scheme [Moulin 80]

A social choice function F on single peaked preference domain $[0, 1]$ is **strategyproof** and **onto** iff it is a **generalized median voter scheme** (GMVS), i.e., there exist 2^n thresholds $\{\alpha_S\}_{S \subset N}$ in $[0, 1]$ such that:

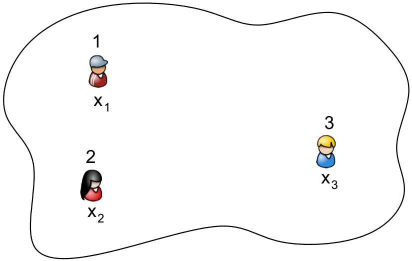
- $\alpha_\emptyset = 0$ and $\alpha_N = 1$ (onto condition),
- $S \subseteq T \subseteq N$ implies $\alpha_S \leq \alpha_T$, and
- for all $(x_1^*, \ldots, x_n^*)$, $F(x_1^*, \ldots, x_n^*) = \max_{S \subset N} \min\{\alpha_S,\ x_i^* : i \in S\}$



k-Facility Location Game

Strategic Agents in a Metric Space

- Set of agents $N = \{1, \dots, n\}$
- Each agent i **wants** a facility at x_i .
  Location x_i is agent i's **private information** .



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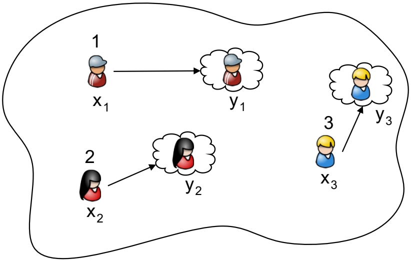
- Each agent i **reports** that she wants a facility at y_i .
  Location y_i may be **different** from x_i.



Mechanisms and Agents' Preferences

(Randomized) Mechanism

A social choice **function** F that maps a location profile $\boldsymbol{y} = (y_1, \ldots, y_n)$ to a (probability distribution over) set(s) of k **facilities** .

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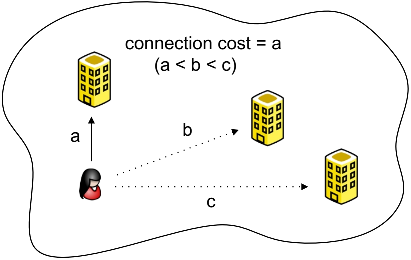
Connection Cost

(Expected) distance of agent i's **true location** to the **nearest** facility:

$$\text{cost}[x_i, F(\boldsymbol{y})] = d(x_i, F(\boldsymbol{y}))$$



Desirable Properties of Mechanisms

Strategyproofness

For any location profile $\boldsymbol{x}$, agent i, and location y:

$$\text{cost}[x_i, F(\boldsymbol{x})] \leq \text{cost}[x_i, F(y, \boldsymbol{x}_{-i})]$$

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Efficiency

$F(\boldsymbol{x})$ should optimize (or approximate) a given **objective function** .

- **Social Cost** : minimize $\sum_{i=1}^{n} \mathrm{cost}[x_i, F(\boldsymbol{x})]$
- Maximum Cost : minimize $\max\{\mathrm{cost}[x_i, F(\boldsymbol{x})]\}$

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- Minimize p**-norm** of $(\mathrm{cost}[x_1, F(\boldsymbol{x})], \ldots, \mathrm{cost}[x_n, F(\boldsymbol{x})])$

1-Facility Location on the Line

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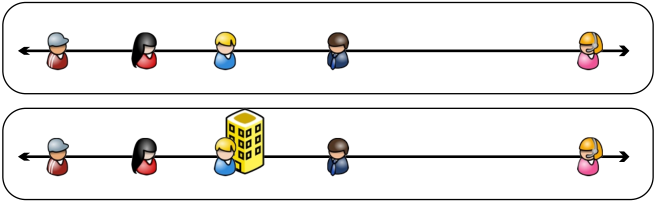
The **median** of $(x_1, \ldots, x_n)$ is **strategyproof** and **optimal** .



1-Facility Location in Other Metrics

1-Facility Location in a Tree [Schummer Vohra 02]

- **Extended medians** are the **only** strategyproof mechanisms.
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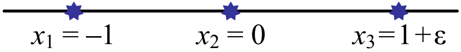
- The optimal solution is **not strategyproof** !
- Deterministic **dictatorship** has cost $\leq (n-1)\mathrm{OPT}$.
- Randomized **dictatorship** has cost $\leq 2\,\mathrm{OPT}$ [Alon FPT 10]

2-Facility Location on the Line

The optimal solution is **not strategyproof** !



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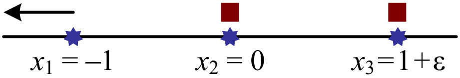

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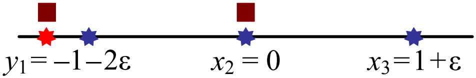

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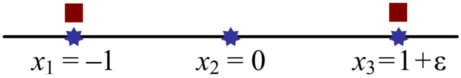
Two Extremes Mechanism [Procacc Tennen 09]

- Facilities at the **leftmost** and at the **rightmost** location :

$$F(x_1, \dots, x_n) = (\min\{x_1, \dots, x_n\}, \max\{x_1, \dots, x_n\})$$

- **Strategyproof** and $(n-2)$**-approximate** .



Approximate Mechanism Design without Money

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- Sacrifice **optimality** for **strategyproofness** .
- **Best approximation** ratio by **strategyproof** mechanisms?
- Variants of k-Facility Location, $k = 1, 2, \ldots$, among the **central** problems in this research agenda.

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2-Facility Location on the Line – Approximation Ratio

	Upper Bound	Lower Bound
Deterministic	$n-2$ [PT09]	$n-2$ [FT12]

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Deterministic	$n - 2$ [PT09]	$n - 2$ [FT12]
Randomized	4 [LSWZ10]	1.045 [LWZ09]

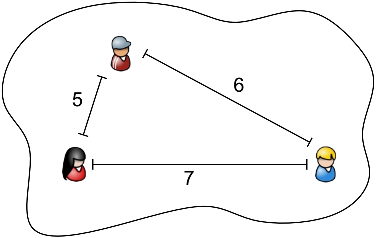
Randomized 2-Facility Location [Lu Sun Wang Zhu 10]

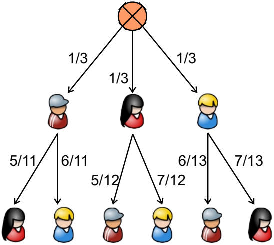
Proportional Mechanism

Facilities open at the locations of **selected agents** .

1st Round: Agent i is selected with probability $1/n$

2nd Round: Agent j is selected with probability $\frac{d(x_j, x_i)}{\sum_{\ell \in N} d(x_\ell, x_i)}$





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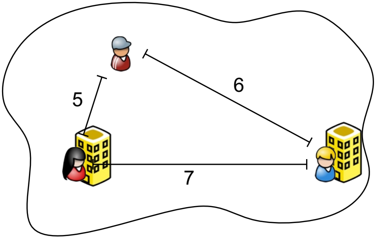

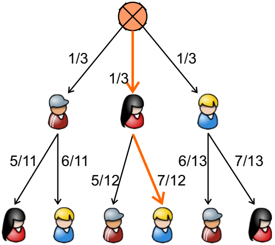

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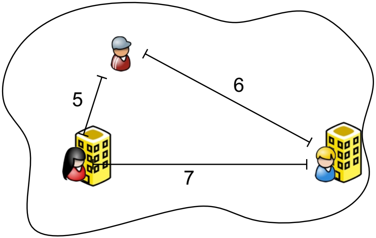
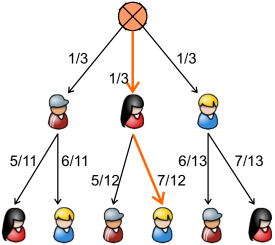
- **Strategyproof** and **4-approximate** for general metrics.
- **Not strategyproof** for > 2 facilities !
  Profile $(0\!:\!\text{many}, 1\!:\!50, 1+10^5\!:\!4, 101+10^5\!:\!1)$, $1 \to 1+10^5$.

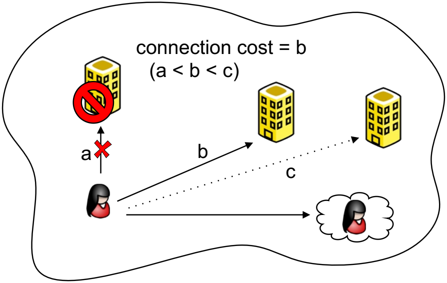




k-Facility Location for $k \geq 3$

Imposing mechanisms

- **Imposing** mechanisms may **penalize liars** by forbidding the agents to connect to certain facilities.
- Agents **connect** to the facility **nearest to reported** location.



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- **Differentially private** mechs are **almost** strategyproof [McSTal 07].
- Complement them with an **imposing gap** mechanism that **penalizes liars** .

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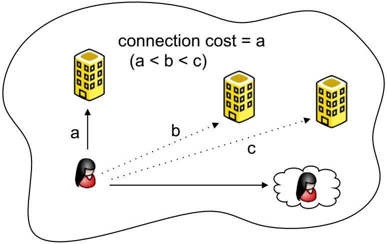
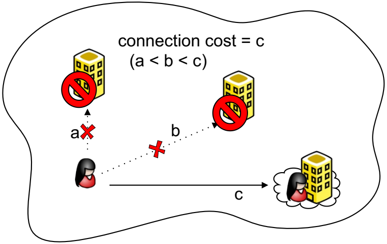
- **Differentially private** mechs are **almost** strategyproof [McSTal 07].
- Complement them with an **imposing gap** mechanism that **penalizes liars** .
- For k-Facility Location on the line, randomized **strategyproof** mechanism with cost $\leq \mathrm{OPT} + n^{2/3}$.
- OPT may be $O(1)$, running time exponential in k.

Winner-Imposing Mechanisms

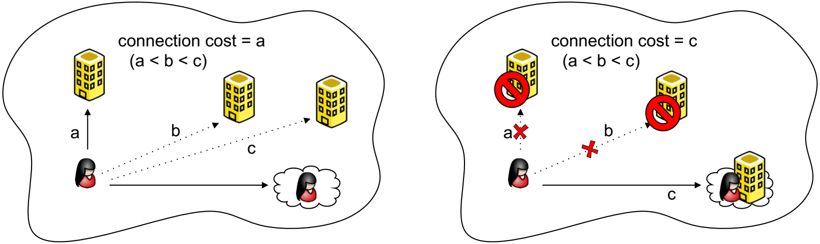
- Agents with a **facility** at their **reported** location **connect** to it. Otherwise, **no restriction** whatsoever.





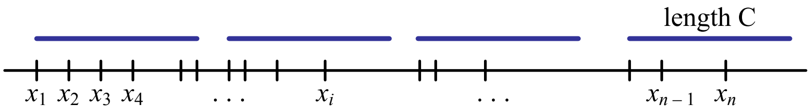
Winner-Imposing Mechanisms

- Agents with a **facility** at their **reported** location **connect** to it. Otherwise, **no restriction** whatsoever.
- **Winner-imposing** version of the Proportional Mechanism is **strategyproof** and **$4k$-approximate** in general metrics, for any k.



Equal-Cost Mechanism

- **Optimal maximum** cost OPT $= C/2$.
- **Cover** all agents with k **disjoint intervals** of length C .



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  With prob. $1/2$, facility at L - R - L - R - ...
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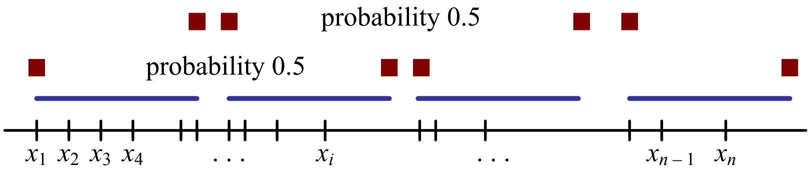

Randomized k-Facility Location on the Line [F. Tzamos 13]

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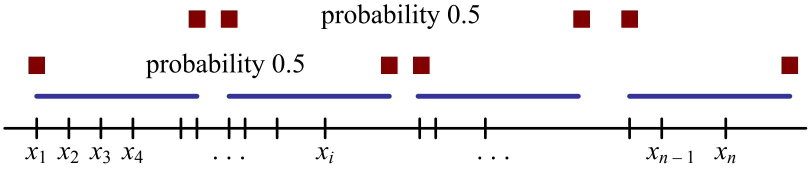
Agents' Cost and Approximation Ratio

- Agent i has expected **cost** $= (C - x_i)/2 + x_i/2 = C/2 = \text{OPT}$.



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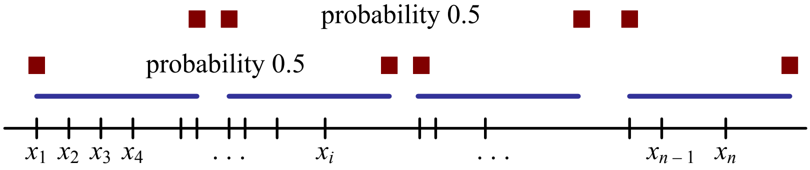
- Agent i has expected **cost** $= (C - x_i)/2 + x_i/2 = C/2 = \text{OPT}$.
- Approx. ratio: 2 for the **maximum** cost, n for the **social** cost.



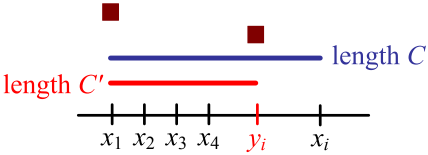
Randomized k-Facility Location on the Line [F. Tzamos 13]

Equal-Cost Mechanism

- **Cover** all agents with k **disjoint intervals** of length C .
- Place a facility to an **end** of each interval .

Strategyproofness

- Agents do not have incentives to **lie** and **increase** OPT.
- Let agent i declare y_i and **decrease** OPT to $C'/2 < C/2$.



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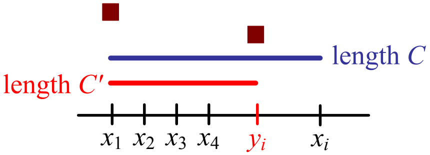

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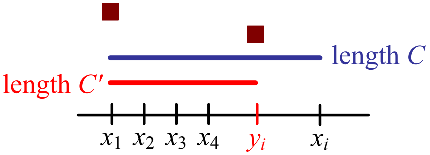
- i's expected **cost** $\geq (C - C')/2 + C/2 = C - C'/2 > C/2$



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Agents with Concave Costs

Generalized Equal-Cost Mechanism is **strategyproof** and has the **same approximation** ratio if agents' cost is a **concave function** of distance to the nearest facility.

Deterministic 2-Facility Location on the Line

Approximation Ratio $\leq n-2$ [PT09]

Place facilities at the **leftmost** and at the **rightmost** location :

$$F(x_1, \ldots, x_n) = (\min\{x_1, \ldots, x_n\}, \max\{x_1, \ldots, x_n\})$$

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Approximation Ratio $> (n-1)/2$ [LSWZ10]

- For all $a < b < 1$, any deterministic **strategyproof** mechanism F with approximation ratio $< (n-1)/2$ must have:

$$F(\underbrace{a, \ldots, a}_{(n-1)/2}, \underbrace{b, \ldots, b}_{(n-1)/2}, 1) = (a, b)$$

- Contradiction for $a = 0$ and $b = 1/n^2$.

Approximability by Deterministic Mechanisms [F. Tzam. 12]

Deterministic 2-Facility Location on the Line

Nice mechanisms ≡ deterministic **strategyproof** mechanisms with a **bounded approximation** .

Niceness **objective-independent** and **facilitates** the characterization!

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Any **nice** mechanism F for $n \geq 5$ agents:

- Either $F(\boldsymbol{x}) = (\min \boldsymbol{x}, \max \boldsymbol{x})$ for all $\boldsymbol{x}$ (Two Extremes).
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Dictatorial Mechanism with Dictator j

- Consider distances $d_l = x_j - \min \boldsymbol{x}$ and $d_r = \max \boldsymbol{x} - x_j$.
- Place the first facility at x_j and the second at $x_j - \max\{d_l, 2d_r\}$, if $d_l > d_r$, and at $x_j + \max\{2d_l, d_r\}$, otherwise.
- **Strategyproof** and $(n-1)$**-approximate** .

Approximability by Deterministic Mechanisms [F. Tzam. 10]

Consequences

- **Two Extremes** is the **only anonymous** nice mechanism for allocating 2 facilities to $n \geq 5$ agents on the line.
- The **approximation ratio** for 2-Facility Location on the line by deterministic strategyproof mechanisms is $n - 2$.

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Deterministic k-Facility Location, for all $k \geq 3$

There are **no anonymous nice** mechanisms for k-Facility Location for all $k \geq 3$ (even on the **line** and for $n = k + 1$).

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Deterministic 2-Facility Location in General Metrics

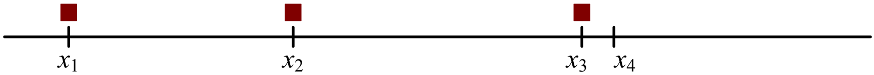
There are **no nice** mechanisms for 2-Facility Location in metrics more general than the line and the circle (even for 3 **agents** in a **star**).

Consistent Allocation for Well-Separated Instances

Well-Separated Instances

- Let F be a nice mechanism for k-FL with approximation ratio ρ.
- $(k+1)$-agent instance $\boldsymbol{x}$ is $(i_1|\cdots|i_{k-1}|i_k, i_{k+1})$**-well-separated** if $x_{i_1} < \cdots < x_{i_{k+1}}$ and $\rho(x_{i_{k+1}} - x_{i_k}) < \min_{2\le\ell\le k}\{x_{i_\ell} - x_{i_{\ell-1}}\}$.

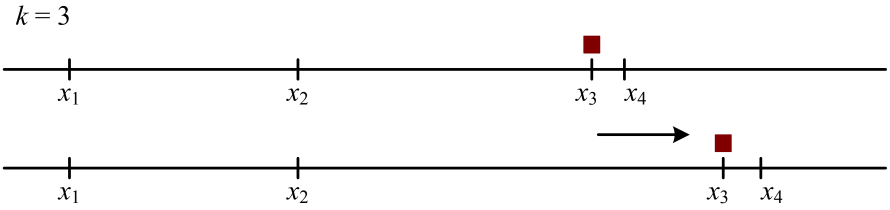
(1|2|3,4)-well-separated instance



Consistent Allocation for Well-Separated Instances

The Nearby Agents Slide on the Right

- Let $\boldsymbol{x}$ be $(i_1|\cdots|i_{k-1}|i_k, i_{k+1})$**-well-separated** with $F_k(\boldsymbol{x}) = x_{i_k}$.
- Then, for all $(i_1|\cdots|i_{k-1}|i_k, i_{k+1})$-well-separated $\boldsymbol{x}' = (\boldsymbol{x}_{-\{i_k, i_{k+1}\}}, x'_{i_k}, x'_{i_{k+1}})$ with $x_{i_k} \le x'_{i_k}$, $F_k(\boldsymbol{x}') = x'_{i_k}$.



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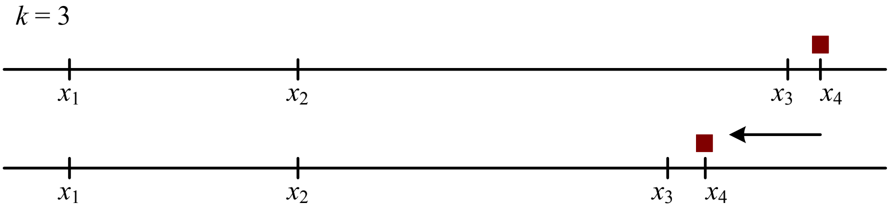

Inexistence of Anonymous Nice Mechanisms for $k \geq 3$

Theorem

There are **no anonymous nice** mechanisms for k-Facility Location for all $k \geq 3$ (even on the **line** and for $n = k + 1$).

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- F **strategyproof** iff all agents get the **best in** their image set .

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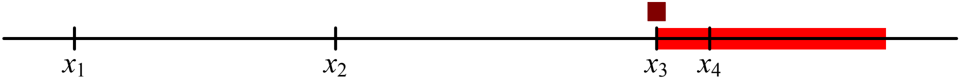

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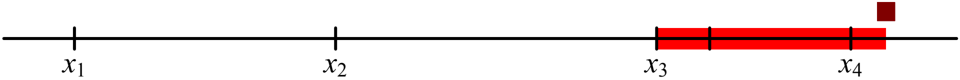

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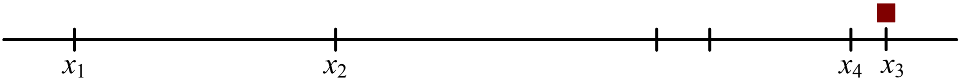

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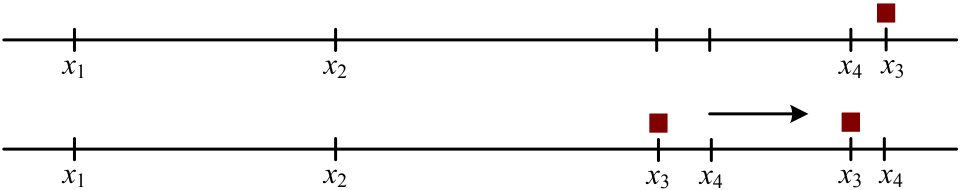
- Contradicts **bounded approximation** ratio of F.



Nice Mechanisms for 2-Facility Location on the Line

Characterization for 3-Agent Instances

Any **nice** mechanism F for $n = 3$ agents:

- $\exists \leq 2$ permutations π_1, π_2 with $\pi_1(2) = \pi_2(2)$: for all $\boldsymbol{x}$ compatible with π_1 or π_2, $\mathrm{med}\, \boldsymbol{x} \in F(\boldsymbol{x})$ (**partial dictator**).
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Well-Separated Instances

Allocation for Fixed Permutation of Nearby Agents

For any agent i and any loc. a, $\exists$ unique **threshold** $p \in [a, +\infty) \cup \{\uparrow\}$: $\forall (i|j,k)$-well-separated $\boldsymbol{x}$ with $x_i = a$,

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- If $p = a$, then $F_2(\boldsymbol{x}) = \max\{x_j, x_k\}$. If $p = \uparrow$, then $F_2(\boldsymbol{x}) = x_j$.

Well-Separated Instances

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For any agent i and any location a,
$\exists$ unique **threshold** $p \in [a, +\infty) \cup \{\uparrow\}$
and **preferred agent** $\ell \neq i$:
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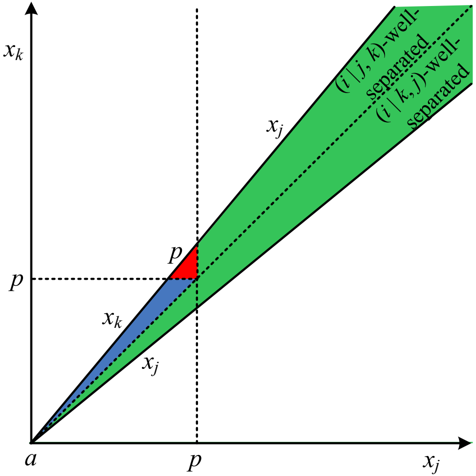
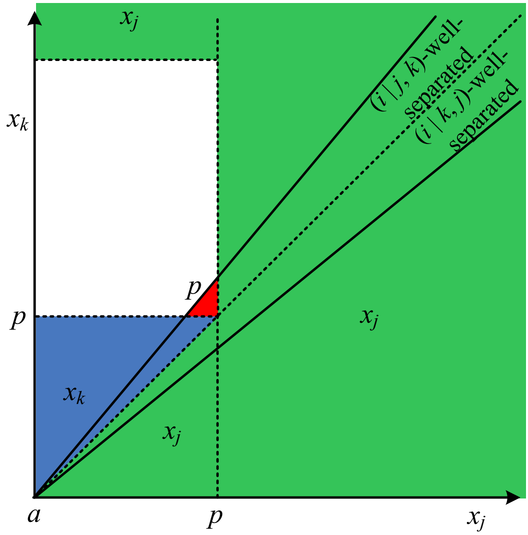


Extension to General Instances



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The Threshold Can Only Take Two Extreme Values

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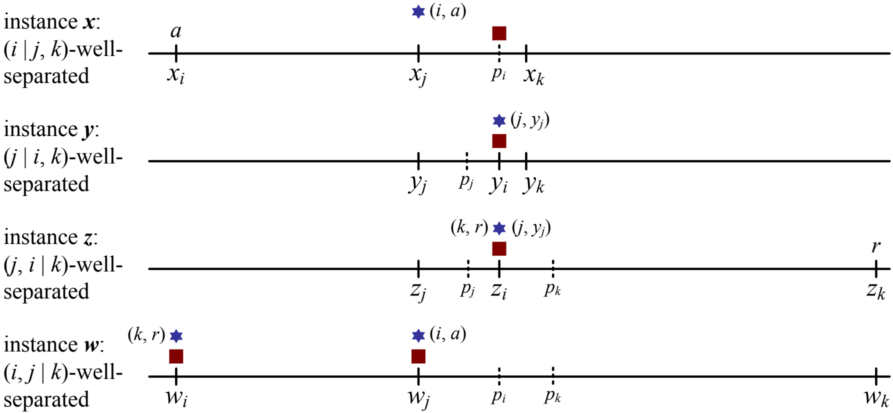

Nice Mechanisms for 2-Facility Location on the Line

Characterization for 3-Agent Instances

Any **nice** mechanism F for $n = 3$ agents:

- $\exists \leq 2$ permutations π_1, π_2 with $\pi_1(2) = \pi_2(2)$: for all $\boldsymbol{x}$ compatible with π_1 or π_2, $\text{med}\, \boldsymbol{x} \in F(\boldsymbol{x})$ (**partial dictator**).
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Characterization for 3-Location Instances

Any **nice** mechanism F for $n \geq 5$ agents on 3 locations:

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- Or admits a unique **dictator** j, i.e., $x_j \in F(\boldsymbol{x})$ for all $\boldsymbol{x}$.

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General Characterization

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- Lower bound of **2** for mechanisms **restricted** to agents' locations.
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  - **Winner-imposing** : lies that increase mechanism's cost cause a (proportional) **penalty** to the agent [F. Tzamos 10] [Koutsoupias 11]
- **Non-symmetric** verification: **conditions** under which the mechanism gets some **advantage** .

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Non-Symmetric Verification to Particular Domains

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- Choice of agents, implementation, what if an agent caught lying?