# CTR + CBC-MAC (CCM)

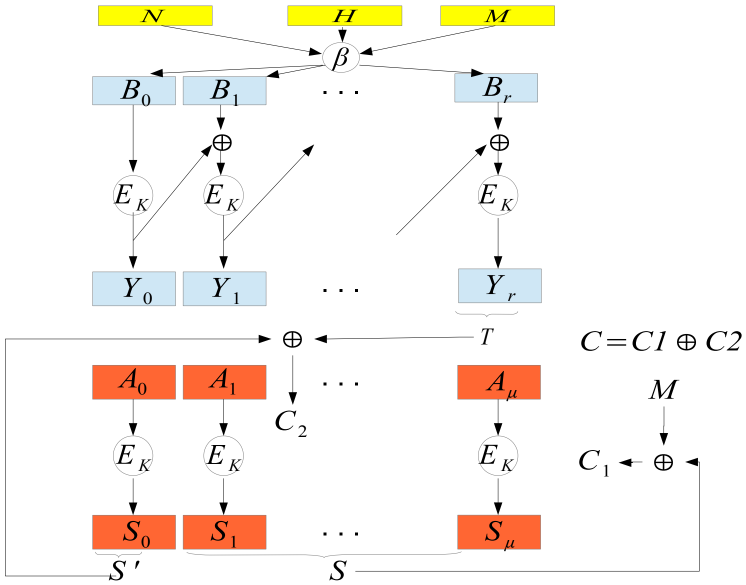
Petros Potikas
NTUA
Number Theory and Cryptography

# Intro

- About block cipher and combined modes
- Authenticated encryption
- Nonce-using symmetric encryption scheme

# Attractive properties

1. Certain parts of message are authenticated only and not encrypted.
2. The underlying block cipher is used only in the forward "encryption" direction, e.g. an arbitrary pseudo-random function can be used.
3. Uses well-known technologies (+20 years)
4. All intellectual property rights are public released.




# Description

- CTR + CBC-MAC provides privacy and authenticity
- CCM is based on a pseudo-random function

$$E : \{0,1\}^{k_0} \times \{0,1\}^{k_b} \rightarrow \{0,1\}^{k_b}$$

- $k_0$ : key bit length
- $k_b$ : block bit length

# Overview

- The parties exchange a secret key, chosen uniformly at random from the set $\{0,1\}^{k_0}$

Input: $(N, H, M)$

- $N$: *nonce*, fixed bit length $k_n < k_b$
- $H$: *header*, only authenticated, not encrypted
- $M$: *message*, authenticated and encrypted
- An authentication tag $(k_t \leqslant k_b)$ is derived via CBC-MAC and encrypted together with the message via CTR

Output: A ciphertext of length $(|M| + k_t)$

## Overview (cont'd)

- Nonce is non-repeating ("fresh") during the lifetime of a key
- Restrictions on input:
  - Lengths of header and message might be upper-bounded by a constant
  - A multiple of 8 or the block length

*V* is the set of all *valid* inputs $(N, H, M)$, i.e. satisfying all requirements.

## CBC-MAC computation

- Encoding function $\beta$

$$\beta : V \rightarrow W^*$$

where $W = \{0,1\}^{k_b}$

- Output: $B = B_0 . B_1 . \cdots . B_r$
- A tag $T$ is derived by applying CBC-MAC to these blocks.
- $B_0$ is the CBC-MAC pre-IV

## CBC-MAC computation

Encoding function $\beta$ must satisfy the following:

- $N$ is uniquely determined by $B_0$
- $\beta$ is prefix-free: for any two valid and distinct inputs $(N, H, M)$ and $(N', H', M')$ with $B = (N, H, M)$ and $B' = (N', H', M')$ $(|B| \leqslant |B'|)$, the string consisting of the first $|B|$ bits of $B'$ does not coincide with $B'$

## CTR encryption

- Encrypt the message $M$ and the CBC-MAC tag $T$.
- Use a CTR block-generator $\pi = (i, N, H, |M|)$ such that $N$ and counter $i$ can be uniquely determined by the CTR block generator.
- $N \in \{0,1\}^{k_n}$ and $0 \leqslant i \leqslant \mu_{max}$ where $\mu_{max}$ is scheme-specific parameter, determine the maximum number of message blocks $number\ of\ blocks \leqslant \mu_{max} \cdot 2^{k_n}$

## CTR encryption

Input (N,H,M), generates input blocks of CTR

$$A_i = \pi(i, N, H, M)$$

$k_t$ leftmost bits of $E_K(A_0)$ are used for encryption of the tag, while the $|M|$ leftmost bits of the string $E_K(A_1).E_K(A_2).E_K(A_3).\cdots$ are used for the encryption of the message $|M|$

## CTR encryption

- Let $\beta_0(N, H, M)$ be equal to the first block $B_0$ of $\beta(N, H, M)$. We require that

$$\pi(i, N, H, M) \neq \beta_0(N', H', M')$$

for all valid (N,H,M), (N',H',M') and $0 \leqslant i \leqslant \mu_{max}$

- The nonce being non-repeating implies that all CTR input blocks $A_i$, and all CBC-MAC pre-IV's $B_0$ used during the lifetime of a key are distinct.

## CCM Specification

- CBC-MAC computation:
  - Let $B_0.B_1.\cdots.B_r = \beta(N, H, M)$
  - Let $Y_0 = E_K(B_0)$
  - For $0 \leqslant i \leqslant r$, let $Y_i = E_K(Y_{i-1} \oplus B_i)$
  - Let T be equal to the $k_t$ leftmost bits of $Y_r$

- CTR encryption:
  - Let $\mu = \lceil |M| / k_b \rceil$
  - For $0 \leqslant i \leqslant \mu$, $A_i = \pi(i, N, H, M)$
  - For $0 \leqslant i \leqslant \mu$, $S_i = E_K(A_i)$
  - Let $S$ be equal to the $|M|$ leftmost bits of $S_1.\cdots.S_m$ and $S'$ the $|T|$ leftmost bits of $S_0$
  - Let $C = [M \oplus S].[T \oplus S']$

## CCM Specification (Decryption)

CCM decryption of a ciphertext $C$ with nonce $N$ and header $H$:

1. Apply the reverse of step 2 to $C$ to obtain a message $M$ and a CBC-MAC tag $T$ (the CTR block generator is applied on $(N, H, |C| - k_t)$.
2. Apply CBC-MAC to the obtained message $M$ to get a tag $T'$. If $T = T'$, then $T'$ is valid, and $M$ is output. Otherwise, $C$ is not valid, and an error is output.

Note: The decryption operation must not release the message or any part of it, until the the tag has been verified.

## Example

- Block cipher: AES, proposed in IEEE 802.11
- Block length $k_b = 128$
- Key length, $k_0 = 128, 192, 256$
- All strings are of length a multiple of 8
- $k_t = 32(16)...128$
- $k_n = 56(8)...112$
- Number of octets in a message $\leqslant 2^{120-k_n} - 1$
- $k_{max} = 120 - k_n$, $\mu_{max} = 2^{k_{max}} - 4$
- Each block contains $2^4$ octets
- Input is valid if $N \in \{0,1\}^{k_n}, 0 \leqslant |H|/8 < 2^{16}, 0 \leqslant |M|/8 < 2^{k_{max}}$

## Example (cont'd)

$$B_0 = (0b)_2.(k_t/16-1)_3.(k_{max}/8-1)_3.(N)_{k_n}.(|M|/8)_{k_{max}}$$

$b=0$, if H is the empty string, 1 o.w. and the two leftmost octets of $B_1$, are equal to $(|H|/8)_{16}$.

- Let $L_H = (|H|/8)_{16}$ if $|H| > 0$, o.w. the empty string then

$$\beta(N, H, M) = B_0.L_H.H.Z_1.M.Z_2$$

where $Z_1$ and $Z_2$ are strings with zeros, such that $|L_H.H.Z_1|$ and $|M.Z_2|$ are multiples of the block length 128.

## Example (cont'd)

- $N$ is uniquely determined by $B_0$
- $\beta$ is prefix free
- Input $(N, H, M)$ is uniquely determined by $\beta(N, H, M)$, because of the inclusion of the octet length of $H$ and $M$

## Example (cont'd)

- The CTR block generator is defined as

$$\pi(i, N) = (00000)_5.(k_{max}/8-1)_3.(N)_{k_n}.(i)_{k_{max}}$$

This cannot be equal to a CBC-MAC pre-IV $B_0$; the first five bits in $B_0$ are not all equal to zeros, since $(k_t/16-1)$ is nonzero.

## *Security analysis - Privacy*

Goal of the adversary: distinguish the ciphertexts from random gibberish (a bit string chosen uniformly at random from the set of all possible bit strings of a specified length)

N is required to be fresh, so CTR input blocks and the CBC-MAC pre-IV's are new and distinct.

The output ciphertext is very close to random gibberish even if the adversary knows the plaintext.

## *Security analysis - Privacy*

Two attacks:

I. A "birthday" attack: All input blocks of CTR are distinct, so no collisions appear among the output blocks, but true random gibberish will contain block collisions, with probability $O(q^2)2^{-b}$ ($q$ number of applications of the block cipher)
II. An anomaly can occur inside the CBC-MAC computations (e.g. an internal collision or a tag to coincide with some CTR output block).This happens with probability $O(q^2)2^{-b}$

## *Security analysis - Authenticity*

Possible to tell anything non-trivial about internal block of the CBC-MAC computation has probability $O(q^2)2^{-b}$ even if all plaintexts are known.
Unless $q$ is very large, the adversary knows close to nothing about the internal block, so modifying any previous encryption query, results in unpredictable modification of the tag.
As $\beta$ is prefix-free, any forgery attempt $B_0.B_1.\cdots$, is unique. If there is a previous encryption query, then they must differ at some point.

Guess the tag: probability less than $2^{-k_t}$